# METHODS

# Calculation of the Liquor System Pliability Using the Mathematical Simulation Method

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Translated from *Byulleten' Eksperimental'noi Biologii i Meditsiny*, Vol. 151, No. 6, pp. 718-720, June, 2011
Original article submitted February 4, 2010

Liquor circulation is a directed flow of the liquor from sites of its secretion to sites of resorption. This slow flow is modulated by pulsation caused by heart work. Phase-contrast magnetic resonance imaging is a method for noninvasive measurements of the linear velocity of these pulses in the cerebral aqueduct. A mathematical model reproducing pulsed flow of the liquor in the cerebral aqueduct is proposed and the procedure of evaluation of these parameters is presented. The pliability liquor system can be calculated from the values of liquor flow linear velocity in the cerebral aqueduct , measured by noninvasive method.

**Key Words:** *liquorodynamics; phase contrast magnetic resonance tomography; liquor system pliability; mathematical simulation*

Liquorodynamics includes secretion of the liquor, its directed movement from sites of secretion to sites of resorption (liquor circulation), and resorption of the liquor [1]. Pulsations caused by heart work are superimposed on this slow flow. Speaking about liquorodynamics, we mean pulsed liquor flow. Liquor flow in the cerebral aqueduct has been actively studied by phase-contrast magnetic resonance imaging (PC MRI) over two recent decades [4,6]. Based on the two-camera model of human liquor system, we carried out mathematical simulation of liquor flow in the cerebral aqueduct. This model took into account such an important parameter as pliability characterizing elastic properties of the liquor system [4]. Evaluation of the liquor system pliability in practical work is the key aspect in the management of hydrocephalic patients.

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We developed a method for calculating pliability in communicating hydrocephaly on the basis of PC MRI data, a noninvasive method for studies of the liquor flow of the cerebral aqueduct.

## MATERIALS AND METHODS

The liquor flow in the cerebral aqueduct was studied by mathematical simulation. The plotting of the mathematical model was based on some theoretical and experimental prerequisites [3,5]. It was assumed within the framework of the two-camera model of the liquor system (Fig. 1) that camera I corresponding to the skull cavity was rigid, while camera II corresponding to the spinal canal possessed certain elasticity properties characterized by pliability *C*. Exchange of a certain volume of the cerebrospinal fluid between the skull cavity and spinal canal cavity was realized during a cardiac cycle. The velocity of liquor passage in the cerebral aqueduct was evaluated. The processes of liquor formation, resorption, and circulation were

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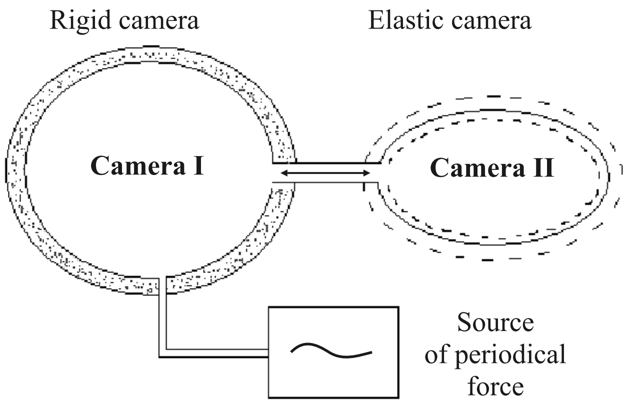


**Fig. 1.** Scheme of two-camera model of the liquor system.

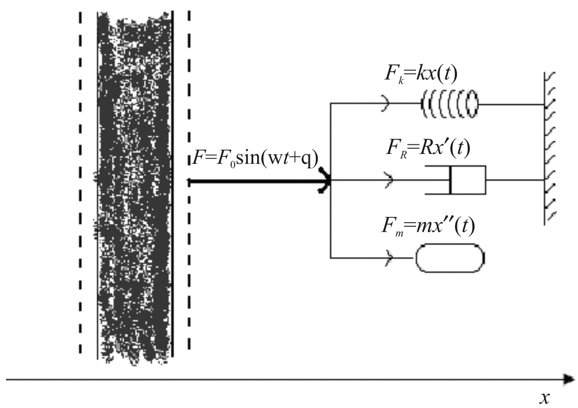


**Fig. 2.** Scheme illustrating the derivation of the main differential equation of the model. $F$: arterial vessel pulsation force, modulating the transported volume of the liquor via the cerebral aqueduct; $F_k$: constituent of $F$ force, directed at stretching of elastic walls of camera II; $F_R$: constituent of $F$ force, directed at overcoming the hydraulic resistance of the aqueduct; $F_m$: constituent of $F$ force, directed at overcoming the inertia of the transported liquor volume weight; $R$: hydraulic friction coefficient; $k$: elasticity coefficient of camera II walls; $m$: weight of the liquor moved via the aqueduct; $x(t)$: movement along axis $x$.

much slower and were therefore neglected. A second-order differential equation was derived and analytically solved [4].

## RESULTS

It has been assumed that force of pulsation of arterial vessels ($F$) acting on the volume of the liquor transported along the cerebral aqueduct (axis $x$) has a sinusoidal pattern:

$$F=F_0\sin(\omega t+\theta). \qquad (1)$$

This force is directed at overcoming the hydraulic resistance of the aqueduct, stretching of the elastic walls of camera II, and overcoming the inertia of the weight of the transported volume of the liquor [4]:

$$F_0\sin(\omega t+\theta)=Rx'(t)+kx(t)+mx''(t), \qquad (2)$$

where $R$ is hydraulic friction coefficient, $k$ is elasticity coefficient of camera II walls, $m$ is weight of the liquor moved in the aqueduct, and $x(t)$ is displacement along axis $x$.

In order to pass from movements along axis $x$ to linear velocity, let us differentiate both parts of the equation:

$$\omega F_0\cos(\omega t+\theta)=R\upsilon'(t)+k\upsilon(t)+m\upsilon''(t), \qquad (3)$$

where $\upsilon$ is the linear velocity of the liquor.

The resultant second-order differential equation was solved analytically. The following function served as its solution:

$$\upsilon(t)=\frac{F_0\sin(\omega t+\theta)}{\sqrt{R^2+(\omega m-\frac{1}{C\omega})^2}}, \qquad (4)$$

where $C=1/k$ ($C$ was liquor system pliability).

Graphically this function was a sinusoid with a period corresponding to the length of one cardiac cy-

**TABLE 1.** Identification of Parameters of Mathematical Model

| Parameter | Value | Estimation procedure |
|---|---|---|
| $\omega$ – cyclic frequency | 6.28 Hz at HR 1 Hz | $\omega=2\pi\upsilon$ |
| C – liquor system pliability | 0.8 ml/mm Hg | [7] |
| m – weight of liquor moved via the aqueduct | 0.06 g | Corresponds to stroke volume for cerebral aqueduct [1] |
| R – hydraulic friction coefficient | 0.028 | Dimensionless value, calculated by formula: $R$=64/Re, where Re is Reynolds' number |
| $\theta$ – phase delay | 2 | |
| $F_0$ – force of pulsation of arterial vessels acting on the liquor volume moved via the aqueduct | 0.5 H | Calculated as the unknown in identity **(3)** with other parameters known and $\upsilon$=4 cm/sec |

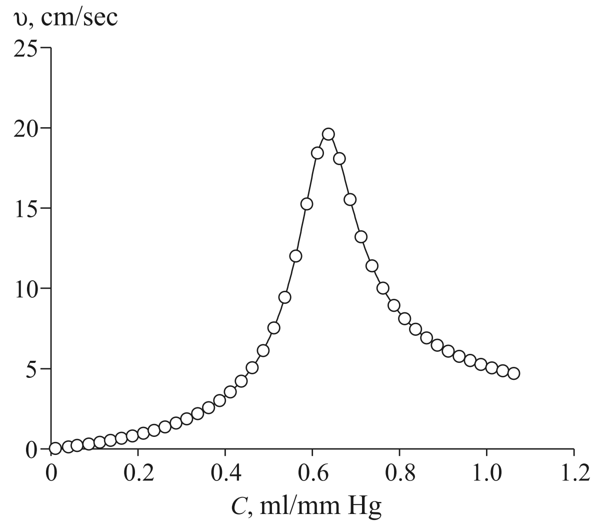


**Fig. 3.** Theoretical relationship between linear velocity of the liquor in the cerebral aqueduct and the liquor system pliability.

cle, while the amplitude was determined by its parameters $R$, $C$, and $F_0$. Then the parameters were identified: some of them were taken from published sources, others were calculated (Table 1).

After substitution of the values for the parameters, the mathematical model can be used for quantitative studies of the liquor flow in the cerebral aqueduct. For example, the function **(4)** obtained by solving differential equation **(3)** relates υ and pliability $C$, an important parameter of the liquor system. If the independent variable $t$ (time) is fixed and the $C$ values modified, the relationship between υ and $C$ is derived (Fig. 3).

From normal υ values (3-5 cm/sec) [3,4] we see that normal values for $C$ are within the interval 0.8-1.0 ml/mm Hg. These estimated values are in a satisfactory agreement with the results of $C$ measurements under conditions of acute experiments (infusion and effusion tests): 1.1-1.2 ml/mm Hg [4,7]. The difference can be attributed to the fact that we neglected pliability of the skull cavity, assuming that camera I walls were absolutely rigid and neglecting actual craniospinal pliability (in fact, we evaluated spinal pliability). The pliability is reduced in hydrocephaly [3,7]. In practical studies, the pliability is often the key parameter in the management of hydrocephalic patients. The individual estimated curve for the patient makes it possible to evaluate the pliability reduction by the υ values and to predict the time course of its reduction with the progress of hydrocephaly.

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